ISSN:2229-6107

INTERNATIONAL JOURNAL OF PURE AND APPLIED SCIENCE & TECHNOLOGY

E-mail :
editor.ijpast@gmail.com
editor.ijpast@.in

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INTERNATIONAL JOURNAL OF PURE AND APPLIED SCIENCE & TECHNOLOGY
IJPAST

ISSN 2229-6107 www.ijpast.in

Vol 9,Issuse 2.May 2019

The correlation between the condition number of a generalised inertia matrix for a serial chain and its aspect ratio

M.BABURAO,K. SRINU,M.SANTHOSH KUMAR,syedsarfarazahmed

Abstract

One way to quantify the degree to which a serial chain is in poor condition is to look at its Generalized Inertia Matrix (GIM) condition number. However, it is computationally costly to calculate the condition number. Therefore, this research looks at several methods of estimating the condition number, especially for a very long serial-chain. This is done by looking at the GIM's diagonal components. Scaled using an initial estimate of the condition number, the ratio of the GIM's greatest and smallest diagonal elements is found to closely reflect the condition number. This greatly streamlines the procedure for identifying GIM MA conditioning, which might be used to determine the system's stability.

. Introduction

For modelling and control purposes, the GIM of a multibody system is a crucial function of its joint variables. Specifically, inadequate training of the GIM leads to errors in forward dynamics [1] and subpar joint control ability [2]. That's why the condition number is such a useful tool for assessing the degree to which the GIM is in poor shape [2]. The condition number is the ratio of the greatest and lowest singular values if norm-2 definition [3] is applied. Since the GIM is symmetric and positive-definite, its condition number is just the ratio of its largest and smallest eigenvalues [3]. However, computing the condition number using eigenvalues is computationally highly costly, despite the fact that it is extensively used as a means to measure ill-conditioning of the GIM.Thereby, huge computational savings may be realized if the GIM's health can be determined using any other attribute of the GIM. To the best of our knowledge, such non-traditional methods were seldom mentioned in published works. It is interesting to note that the pivot ratio, defined as the ratio of the largest to the smallest pivot element, may be used to foretell instability, as shown by Mitra and Klein [4] in 1975. They implemented the idea in electromagnetics integral equations. Nonetheless, it became clear throughout the course of this investigation that pivot cannot be relied upon to accurately predict the trajectory of the condition number. We use the ratio of the largest to smallest diagonal elements of the GIM as a measure of ill-conditioning of the GIM because 1) the trace of GIM is equal to the sum of eigenvalues, and 2) each diagonal element of the GIM carries

ASSISTANT PROFESSOR[1,2,3],STUDENT [4]
Department of MECH
Arjun College Of Technology & Sciences
Approved by AICTE& Affiliated to JNTUH

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the knowledge of the system ahead of the body corresponding to the index of the diagonal element. For the purpose of brevity, we shall refer to this proportion as the diagonal ratio from here on. The GIM's constituents are easily accessible as a by-product of either inverse or forward dynamics techniques [1], therefore the diagonal ratio may be computed without any further work. We used the diagonal ratio to condition number correlation to determine poor conditioning. Scaled diagonal ratio is then implemented later on. The remainder of the paper is laid out as follows: Section 2 introduces GIM ill-conditioning. Section 3 introduces the GIM and its key features, while Section 4 offers a number of numerical examples. Conclusions are presented in Section 5.

Preparation of the GIM

As stated in [5], a tree-structured multibody system's equations of motion may be modelled as a

$$\mathbf{I}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} = \boldsymbol{\tau}$$

When GIM (represented by I), a generalized coordinate vector (generalized coordinates), a matrix of convective inertia components (represented by C), and a generalized external force vector (represented by) are all given. A poorly-conditioned GIM may have repercussions on a system's control performance, as shown in [2]. Therefore, the ill-conditioning metric may be useful for making adjustments in simulation or control. However, this is beyond the purview of the research; instead, we concentrate on efficient estimate of ill-conditioning, which may be used as a reference. Here, we merely utilize simulation to show how the GIM degrades with time.

Solving a set of linear algebraic equations in joint accelerations is the first step in simulating a multibody system. The second step is numerical integration. The q-valued joint accelerations are calculated using Eq. (1).

$$\ddot{\mathbf{q}} = \mathbf{I}^{-1}\boldsymbol{\varphi}, \text{ where } \boldsymbol{\varphi} = \boldsymbol{\tau} - \mathbf{C}\dot{\mathbf{q}}$$

Since the GIM may be factorized using LU or Cholesky decomposition [3], and then q is computed via backward and forward replacements [3], it is not necessary to do an explicit inversion of the GIM, I, in order to solve for q. When the GIM is ill-conditioned, however, even moderate changes in the system's solutions might have a significant impact. An ill-conditioned matrix is one that is very near to its singularity [3]. The solution is perturbed by even a tiny modification in the right-hand side of Eq. (2).

$$(\ddot{\mathbf{q}} + \boldsymbol{\delta}\ddot{\mathbf{q}}) = \mathbf{I}^{-1}(\boldsymbol{\varphi} + \delta\boldsymbol{\varphi}),$$

Resulting in the relative error [3]

$$\frac{||\boldsymbol{\delta}\ddot{\mathbf{q}}||}{||\ddot{\mathbf{q}}||} \leq \left\|\mathbf{I}^{-1}\right\| ||\mathbf{I}|| \frac{||\boldsymbol{\delta\varphi}||}{||\boldsymbol{\varphi}||}$$

where the GIM norm is denoted by kick, and (I) = I 1 The condition number kIk of an equation is the multiplier applied to the answer q as a result of a modest change in the right-hand side. If the GIM has a high condition number, it is poorly conditioned or on the brink of singularity. The GIM condition number is determined from if we choose norm-2 [3].

$$\kappa_2(\mathbf{I}) = \frac{\sigma_{max}(\mathbf{I})}{\sigma_{min}(\mathbf{I})}$$

, with the maximum and lowest singular values of the GIM denoted by max(I) and min(I), respectively. Due to the GIM's symmetry and positive definiteness, its singular values are equivalent to the eigenvalues, and Eq. (5) may be rewritten as

$$\kappa_2(\mathbf{I}) = \frac{\lambda_{max}}{\lambda_{min}}$$

where the largest and smallest eigenvalues of the GIM are respectively denoted by max and min. It is important to note that the worst condition numbers for serial multibody systems with identical links or homogeneous rods are on the order of O (4n 4) [1], where n is the total number of connections. Therefore, there is a considerable probability of accuracy loss in the calculation of the joint accelerations as n increases. In order to offer a reliable solution, a numerical integrator may need to use very tiny step sizes. The term "numerical" might be used to describe this occurrence.

rigidity [6]. Fig. 1 depicts a hypothetical scenario in which a 4-link serial chain composed of identical links is explored, with the chain traveling downwards under the force of gravity to provide some perspective on the phenomenon of poor conditioning. Assume that each link is a homogenous, thin rod of length l = 1m and mass m = 2.2 kg. For the state q = q = 0, we get the GIM I and the forces as

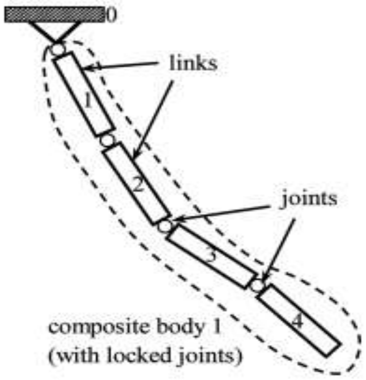


Fig. 1. A 4-link chain with only revolute joints

$$\mathbf{I}=\begin{bmatrix} 47.0822 & & sym & \\ 29.7942 & 19.8629 & & \\ 14.7132 & 10.2993 & 5.8854 & \\ 4.0462 & 2.9427 & 1.8392 & 0.7357 \end{bmatrix}, \text{ and } \boldsymbol{\varphi}=\begin{bmatrix} -86.6015 \\ -48.7133 \\ -21.6504 \\ -5.4126 \end{bmatrix}$$

where "sym" indicates GIM symmetry. Accelerations at the joints may be solved for in the following way:

$$\ddot{\mathbf{q}}=\mathbf{I}^{-1}\boldsymbol{\varphi}=\begin{bmatrix} -6.2189 \\ 7.8864 \\ -2.1226 \\ 0.6071 \end{bmatrix}$$

In Eq. (7), the GIM has a condition number of 2(I) =1074, which is rather large for such a compact system. A small sample size is required for observing the impact of

Rounding errors in are regarded to be perturbations of on q.

$$\boldsymbol{\varphi}=\begin{bmatrix} -86.0 \\ -48.7 \\ -21.6 \\ -5.4 \end{bmatrix}, \text{ resulting in } \ddot{\mathbf{q}}=\mathbf{I}^{-1}\boldsymbol{\varphi}=\begin{bmatrix} -5.7477 \\ 6.7586 \\ -1.2128 \\ 0.2696 \end{bmatrix}.$$

Thus, the very large percentage changes in accelerations q1, q 2, q 3, and q4 of 7%, 14%, 42%, and 55% are the consequence of relatively modest percentage changes in 1, 2, 3, and 4 of 0.69%, 0.02%, 0.23%, and 0.23%, respectively. With larger systems, this shift will be much more noticeable. As a result, it is crucial to have a good assessment of how badly conditioned the GIM is in the first place. In [5, 7], it was shown that the GIM, developed by using the idea of the Decoupled Natural Orthogonal Complement (DeNOC) matrices, retains the data of mass and inertia characteristics in a fairly systematic way. Therefore, the GIM's components might provide in-depth knowledge about poor conditioning with further investigation. As a result, we'll move on to a discussion of the GIM's most salient features.

The GIM's Distinctive Features

Following is a representation for a serial chain's GIM.

$$\mathbf{I}\equiv\begin{bmatrix} I_{11} & & & & sym \\ I_{21} & I_{22} & & & \\ I_{31} & I_{32} & I_{33} & & \\ \vdots & \vdots & \ddots & \ddots & \\ I_{n1} & I_{n2} & \cdots & I_{nn-1} & I_{nn} \end{bmatrix}.$$

In this case, Iij is the (i, j) the GIM element. One may use [7] as an analytic expression for the (i, j) the element of the GIM.

$$I_{ii}=\mathbf{p}_i^T\tilde{\mathbf{M}}_i\mathbf{p}_i$$
$$I_{ij}=\mathbf{p}_i^T\tilde{\mathbf{M}}_i\mathbf{A}_{i,j}\mathbf{p}_j\equiv I_{ji}$$

For example, consider Eq. (11), where Ai, j and pj are the twist-propagation matrix and motion propagation vectors, respectively [7], and M I is the mass matrix of composite body, which contains the mass and inertia properties of the system consisting of all rigidly connected links upstream of the I the link, including itself. This is calculated using the mass matrix Mi of the itch connection.

$$\tilde{\mathbf{M}}_i=\mathbf{M}_i+\mathbf{A}_{j,i}^T\tilde{\mathbf{M}}_j\mathbf{A}_{j,i}$$

where for the terminal link M˜ n = Mn. The structure of the mass matrix of a composite body may vary with the choice of independent generalized coordinates. However, the present choice is based on a popular choice for the serial-type systems, i.e., relative coordinates. It is worth noting that the GIM is a positive definite matrix, and hence, the diagonal terms are always greater than zero, i.e., p T I M˜ ipi>0 for I =1, . . ., n. Using the analytical expressions in Eq. (11), the GIM of the 4-link planar chain, shown in Fig. 1, is obtained as

$$\mathbf{I}=\begin{bmatrix} \mathbf{p}_1^T\tilde{\mathbf{M}}_1\mathbf{p}_1 & & & sym \\ \mathbf{p}_2^T\tilde{\mathbf{M}}_2\mathbf{A}_{21}\mathbf{p}_1 & \mathbf{p}_2^T\tilde{\mathbf{M}}_2\mathbf{p}_2 & & \\ \mathbf{p}_3^T\tilde{\mathbf{M}}_3\mathbf{A}_{31}\mathbf{p}_1 & \mathbf{p}_3^T\tilde{\mathbf{M}}_3\mathbf{A}_{32}\mathbf{p}_2 & \mathbf{p}_3^T\tilde{\mathbf{M}}_3\mathbf{p}_3 & \\ \mathbf{p}_4^T\tilde{\mathbf{M}}_4\mathbf{A}_{41}\mathbf{p}_1 & \mathbf{p}_4^T\tilde{\mathbf{M}}_4\mathbf{A}_{42}\mathbf{p}_2 & \mathbf{p}_4^T\tilde{\mathbf{M}}_4\mathbf{A}_{43}\mathbf{p}_3 & \mathbf{p}_4^T\tilde{\mathbf{M}}_4\mathbf{p}_4 \end{bmatrix}$$

In Eq. (14), M˜ 4 = M4 represents the mass and inertia properties of the 4 the link only, whereas, M˜ 1 represents the mass and inertia properties of all the links, enclosed by the dotted line in Fig. 1. Therefore, the term p T 1M˜ 1p1 is larger than any other diagonal term and p T 4M˜ 4p4 is the smallest of all. This is also evident from Eq. (7). Moreover, it obvious that with the increase in the number of links, the term p T 1M˜ 1p1 will become larger and larger, whereas p T nm. ˜npn will remain unaffected. Moreover, the 'trace' of the GIM, i.e., the sum of the diagonal elements, is related to the eigenvalues [3] by

$$tr(\mathbf{I}) = \sum_{i=1}^{n} I_{ii} = \sum_{i=1}^{n} \lambda_i, \text{ where } I_{ii} = \mathbf{p}_i^T \tilde{\mathbf{M}}_i \mathbf{p}_i$$

The above two facts motivated us to compare the ratio of the smallest and highest eigenvalues, i.e., condition number in Eq. (6), with the ratio of the largest to smallest diagonal elements of the GIM. These ratios are compared using several numerical examples in the next section.

Numerical Illustrations

As introduced in Section 1, the diagonal ratio is defined as the ratio of the largest to smallest diagonal elements of the GIM and will be denoted as δ(I) = I11/Inn hereafter. The eigenvalues and the diagonal elements of the GIM for the swinging 4-link chain are plotted in Figs. 2(a-b). It can be seen that the element I11 > (I22, I33, I44) follows the trend of the highest eigenvalue λ1 throughout the simulation period. Since, I44 is the smallest element, δ(I) = I11/I44 forms the diagonal ratio, Fig. 2(d). Comparison with the condition

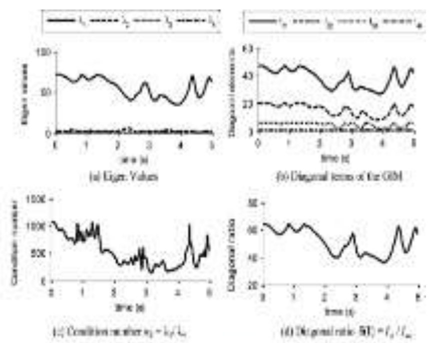


Figure 2: The GIM of a 4-link chain has a variety of features.

Figure 2(c) demonstrates that the diagonal ratio accurately represents the underlying trend of the condition number, given as 2 = 1/4.

$$\delta(\mathbf{I}) = I_{11}/I_{44}$$

Then, Fig. 3 compares 10-link and 20-link serial chains with identical links based on their diagonal ratio and condition number. Figure 3 shows that the maximum condition number and the diagonal ratio both grow as the network size grows. The diagonal ratio may also be used to represent the general trend of the condition number, which is true for both 10- and 20-link chains. Similarly, the worst condition number for a serial-chain with identical links is approximately of O (4n 4) [1], as shown in Figs. 2(c), 3(a), and 3(c). The ratio of the biggest pivot element to the smallest produced from Gaussian elimination [3] of the GIM was shown to be a useful indicator of ill-conditioning, as discussed in [4]. To further illustrate that the pivot ratio does not represent the trend of the condition number, Fig. 4 also displays the pivot ratios of 10- and 20-link chains. As a result, serial-chain systems cannot utilize the pivot ratio to assess the degree to which the GIM is ill-conditioned. While the condition number does track the diagonal ratio, the two magnitudes are still somewhat different. The concept of a scaled diagonal ratio is presented to help get a rough idea of how big the condition number is. It's easy to see why the scaled diagonal ratio is just the diagonal ratio multiplied by a constant.

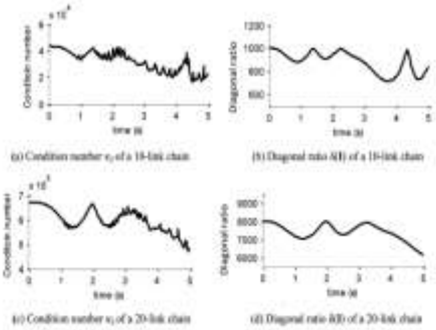


Fig. 3. Comparison of the condition number and diagonal ratios for two serial chains

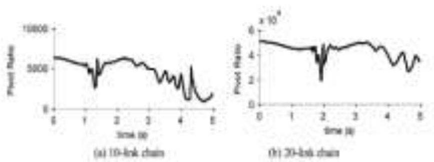


Fig. 4. Pivot ratios for 10- and 20- link chains

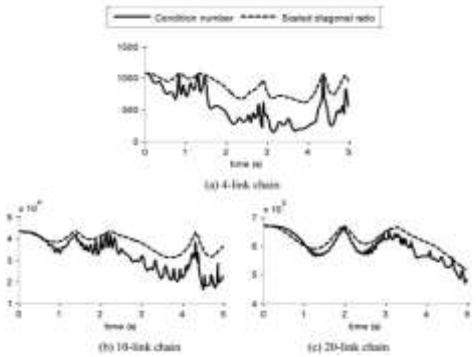
$$\delta_s(\mathbf{I}) = \alpha\delta(\mathbf{I}) \text{ where } \alpha = \frac{\kappa(\mathbf{I})|_{t=0}}{\delta(\mathbf{I})|_{t=0}}$$

condition number of a system at time t=0

According to Fig. 5, s(I) not only accurately predicts the condition number but also accurately captures its trend. This demonstrates that ill-conditioning of the GIM may be estimated via simulation using the scaled diagonal ratio s(I) without the need for costly calculation of the condition number.

Conclusions

In this study, we provide a unique approach to simulating ill-conditioning of the GIM and then estimating its severity. The ratio of the greatest to the smallest is used in this technique.



Condition number and scaled diagonal ratio for 4-, 10-, and 20-link chains in Fig. 5.

Generalized Inertia Matrix (GIM) diagonal elements multiplied by a constant factor. Multiple numerical examples are provided to demonstrate the method's efficacy. To make a highly secure choice regarding the GIM's poor condition, we may use the diagonal ratio, which records the trend of the condition number, and scale it using the beginning values of the condition number, which then gives the magnitude of the condition number. The suggested technique not only simplifies and expedites the estimate of bad conditions, but it also provides a useful tool for future work in improving control performance and defining adaptive tolerances for the forward dynamics issue.

REFERENCES

[1] Featherstone, R.: An Empirical Study of the Joint Space Inertia Matrix. International Journal of Robotics Research, Vol. 23, No. 9, pp. 859-871, 2004.

[2] Shen Y., Featherstone R.: The Effect of Ill-conditioned Inertia Matrix on Controlling Robot Manipulator. Australasian Conference on Robotics and Automation (ACRA), 2003.

[3] Watkins D.S.: Fundamentals of Matrix Computations, Wiley Intercedence, New York, 2002.

[4] Mitra R., Klein C. A.: Use of Pivot Ratios as a Guide to Stability of Matrix Equations Arising in the Method of Moments. IEEE Transactions on Antennas and Propagation, Vol.23, No. 3, pp. 448-450, 1975.

[5] Saha, S. K.: "Dynamics of Serial Multibody Systems Using the Decoupled Natural Orthogonal Complement Matrices," ASME J. of Applied Mechanics, 66, 986-996, 1999.

[6] Sham pine L.F., Gear C.W.: A User's View of Solving Stiff Ordinary Differential Equations. SIAM Review, Vol. 21, No. 1, 1979.

[7] Saha S.K.: Analytical Expression for the Inverted Inertia Matrix of Serial Robots. Int. J. of Robotic Research, Vol. 18, No. 1, pp. 116-124, 1999.